Dynamical systems for remote validation of very high-resolution ocean models

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I. INTRODUCTION

The Earth's complex and inherently turbulent ocean dynamics plays a crucial role in global climate change and many human activities. Gaining insight into ocean currents is especially critical for environmental concerns such as addressing disasters like oil spills ([3–5]), given that these currents influence pollutant dispersion. Moreover, ocean currents significantly contribute to the growing global issue of plastic pollution, ferrying plastic waste found in lands, rivers, oceans, and even Alpine snow to the Arctic [2]. To tackle this issue, a thorough comprehension of the intricate transport processes triggered by ocean movement is necessary, albeit challenging. This understanding is crucial for surveillance and mitigation of the impacts on global climate change and marine resources.

Materials transported via oceanic flows often trace chaotic routes, denoted as $\mathbf{x}(t)$. These can be explicated via a purely advective approach, focusing on the motion of a fluid parcel in the ocean. This approach is based on the solution of an equation that describes the velocity of the fluid parcel as it navigates the ocean:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}, t), \mathbf{x} \in \mathbb{R}^n. \tag{1}$$

This equation, an ordinary differential equation or a dynamical system, postulates that a fluid parcel's velocity at a specific point equates to the ocean current's velocity at the same point. However, examining such systems poses challenges due to the chaotic nature of ocean flows, where proximate fluid particles can trace entirely distinct routes despite seemingly similar flow conditions. Even under controlled lab conditions, fluid parcels trace complex paths, making the study of Lagrangian transport a complicate task [11].

Addressing this issue, researchers have proposed concepts building upon Poincaré's work on dynamical systems theory over recent decades; see, for instance [1, 9, 12]. This approach, known as the dynamical systems approach to Lagrangian transport, aims to identify geometric flow structures that divide the ocean into distinct regions, each corresponding to trajectories with qualitatively different dynamical behaviours. These unique material fluid structures, also known as Lagrangian Coherent Structures (LCS), act as transport barriers that fluid particles cannot traverse, thereby regulating transport and mixing processes between different flow areas. Notably, these tools can unveil a hidden order via geometrical structures. In this regard, Dynamical Systems Theory (DST) has offered a framework for depicting these solutions, and some objectives of this study are precisely aimed at harnessing this information to illustrate selected environmental issues.

While DST offers potent tools for analysing the transport capacity of ocean currents, these models necessitate reliable and accurate ocean current data, which is becoming increasingly accessible. For example, the Copernicus Marine Environment Monitoring Service (CMEMS) furnishes regular, systematic data about the global and European regional seas' physical state and ocean dynamics.

II. DATA AND METHODOLOGY

A. Ocean data

The ocean velocity data utilized in our study to compute the Lagrangian Descriptor were sourced from the Copernicus Marine Environment Monitoring Service (CMEMS). We specifically used datasets from the high-resolution Baltic Sea Physics Analysis and Forecast product https://doi.org/10.48670/moi-00010. This product furnishes forecast data on the physical conditions of the Baltic Sea, developed from a dedicated Baltic Sea physical model. The forecast is refreshed twice a day, producing a six-day predictive outlook. The product includes four distinct datasets: an hourly instantaneous values set, a daily mean values set, a monthly mean values set, and a 15-minute (instantaneous) surface values set. For our research, we utilized the daily dataset, which integrates parameters such as sea level variations, surface ice concentration and thickness, temperature, salinity, and the horizontal and vertical velocities of the 3D field. Our primary focus was on the analysis of sea surface currents.

The forecast product is the outcome of a Baltic Sea configuration of the NEMOv4.0 ocean model, delivered on the model's native grid with a horizontal resolution of 1 nautical mile and up to 56 vertical depth levels. Its geographical coverage spans the entire Baltic Sea, including the transition area towards the North Sea (i.e., the Danish Belts, the Kattegat, and Skagerrak).

FIG. 1. Caption

The ocean model incorporates Stokes drift data from the Baltic Wave forecast product (BALTICSEA_ANALYSISFORECAST_WAV_003_010 https://doi.org/10.48670/moi-00011). Furthermore, the model's analysis field absorbs satellite sea surface temperature (SST) data and in-situ temperature and salinity profiles.

In our research, we have also made use of the FjordOS CL model developed by [10]. The FjordOs CL has a spatial grid size that ranges from approximately 50 meters in the Drøbak sound to about 300 meters at its southern open boundary adjoining the Skagerrak. The model incorporates 42 terrain-following levels vertically. It's influenced by diverse factors including atmospheric, river, and tidal input, as well as oceanic input at the open boundary. Atmospheric input is derived from MET Norway's operational Numerical Weather Prediction (NWP) model, AROME-MetCoOp, while the oceanic input comes from MET Norway's operational ocean forecasting model, NorKyst800. The model also considers river input consisting of observational-based estimated discharges from 37 rivers along the fjord's perimeter. The tidal input, sourced from the TPXO Atlantic database and refined by observations, comprises nine tidal constituents. This all-encompassing model has been instrumental in our work, providing a well-rounded and nuanced perspective on our research objectives.

B. Lagrangian Descriptors

The Lagrangian descriptor, initially known as the M function, was introduced in [6] and subsequently investigated in the realm of real ocean flow [8]. This research showcased its potential to offer a comprehensive understanding of the geometric structures present in time-dependent flows. In particular, Lagrangian descriptors are able to detect the main "organizing centers" in the flow, including hyperbolic trajectories and their repelling and attracting LCSs, as well as elliptic regions.

We consider a general time-dependent vector field on $\mathbb{R}^n$ as equation 1, where we assume that $\mathbf{v}(\mathbf{x}, t)$ is $C^r (r \geq 1)$ in $\mathbf{x}$ and continuous in t. This is a sufficient condition for the existence of unique solutions that also allows for linearization. In the case of ocean flows this condition is satisfied.

We begin by describing the Lagrangian descriptor denoted by M. M is the Euclidean arc length of the curve in phase space defined by a trajectory of the vector field starting at $\mathbf{x}^*$ at time $t = t^*$ for the time interval $[t^* - \tau, t^* + \tau]$, i.e.,

$$M(\mathbf{x}^*, t^*)_{\mathbf{v},t} = \int_{t^*-\tau}^{t^*+\tau} \sqrt{\sum_{i=1}^{n} \left(\frac{dx_i(t)}{dt}\right)^2} dt = \int_{t^*-\tau}^{t^*+\tau} \|\mathbf{v}(\mathbf{x}(t), t)\| dt, \tag{2}$$

where $(x_1(t), x_2(t), ..., x_n(t))$ denote the components of the trajectory $\mathbf{x}(t)$ in $\mathbb{R}^n$. Clearly, M depends on the initial point $\mathbf{x}^*$ and the time interval $[t^* - \tau, t^* + \tau]$ (and the vector field $\mathbf{v}$).

A heuristic argument can be presented to demonstrate the usefulness of M in uncovering the geometric structures within the phase space of Eq. 1. M quantifies the arc length of trajectories over a time interval $(t^{-}\tau, t^{+}\tau)$. Trajectories originating from initially "close" conditions and maintaining proximity as they evolve during this time interval are expected to exhibit similar arc lengths. However, when approaching the boundaries separating regions with qualitatively different behaviour during the time evolution within $(t^{-}\tau, t^{+}\tau)$, the arc lengths of trajectories originating from either side of the boundary are anticipated to deviate significantly.

Therefore, these boundaries are characterized by a distinct "abrupt change" in M, indicating a discontinuity in the derivative of M transverse to these boundaries. Such boundaries correspond to the stable and unstable manifolds of hyperbolic trajectories or the repelling and attracting Lagrangian Coherent Structures (LCS), respectively, of hyperbolic trajectories. Consequently, M effectively detects the presence of stable and unstable manifolds associated with hyperbolic trajectories. For more comprehensive information and formal proofs, we recommend referring to [7]. In Fig. 1, you can find an example of the M function in the Oslo Fjord using CMEMS Baltic Sea Product.

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